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FREE CONVECTION HEAT TRANSFER IN AN OPEN SYSTEM OF VERTICAL RODS

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Investigations of free convection heat transfer on vertical surfaces are a well-developed section of the theory of natural convection flows. Extensive handbook and bibliographic material on this topic can be found in [1]. However, the problems of developing effective heat transfer apparatus, the necessity to compute the temperature regimes of complex rod systems possessing heat liberation, the selection of effective methods of protecting packets of electrical cables from overheating determine the urgency of formulating and solving problems on the hydrodynamics and heat transfer of different sets of rods. An effective model that permits reflection of the hydrodynamic and thermal interaction of rods between themselves and the bundle as a whole with the environment is a filtration flow model. It is used extensively at this time for heat transfer computations under forced convection in anisotropic rod structures [2-5]. Critical relationships obtained on the basis of processing experimental data are used here to determine the thermal and hydrodynamic forces of solid and liquid phase interaction per unit volume of a porous body. Considerably less attention is paid to questions of mathematical modeling of free convection heat transfer in such media. Existing researches are mainly experimental in nature [6-9]. Consequently, application of the filtration flow model in a porous medium to the description of free convection processes in rod bundles and execution of numerical computations of the heat transfer of rod collections with an external cooling medium are of great interest. Meanwhile the lack of critical dependences for bulk friction and heat liberation in such a flow specifies the urgency of the problem of a theoretical determination of the desired quantities. Solution of these problems is indeed the purpose of this paper.

1. FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

Let us consider the axisymmetric free convective incompressible fluid flow in a vertical bundle of rods. We assume that the flow mode is laminar and the viscosity, heat conductor, and specific heat are independent of the temperature. We direct the x axis along the longitudinal axis of the bundle, then the r axis will lie in a plane perpendicular to the x axis while the angle φ is measured from a certain initial position of the xor plane. Let us extract a small space element $\Delta V = r\Delta\varphi\Delta x\Delta r$, containing a sufficiently large quantity of rods in addition to fluid (Fig. 1). In the presence of the rods the space configuration is characterized by the quantities

$$\varepsilon = \Delta V_f/\Delta V,\quad \varepsilon_x = \Delta S_{fx}/\Delta S_x,\quad \varepsilon_r = \Delta S_{fr}/\Delta S_r,\quad \varepsilon_\varphi = \Delta S_{f\varphi}/\Delta S_\varphi,$$

where ΔV_f is the volume of space occupied by the fluid, ΔS_j is the area of a side of the element with normal along the appropriate axis, and ΔS_{fj} is the area of the flow-through part of the appropriate side of the element. The flow field is determined by the velocity vector $\mathbf{V} = \mathbf{i}u + \mathbf{j}v$ as well as by mass and volume force vectors acting on the extracted element. Taking into account the axisymmetry of the motion and using the standard procedure for deriving the conservation equations for a continuous medium [10], we have

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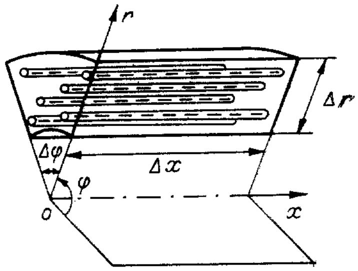

Fig. 1

$$\varepsilon_x \partial ur/\partial x + \varepsilon_r \partial vr/\partial r = G,$$

$$\varepsilon \frac{\partial u}{\partial \tau} + \varepsilon_x u \frac{\partial u}{\partial x} + \varepsilon_r v \frac{\partial u}{\partial r} = R_x - \frac{\varepsilon_x}{\rho} \frac{\partial p}{\partial x} + A\varepsilon + \nu_{ef}\left[\varepsilon_x \frac{\partial^2 u}{\partial x^2} + \frac{\varepsilon_r}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right)\right],$$

$$\varepsilon \frac{\partial v}{\partial \tau} + \varepsilon_x u \frac{\partial v}{\partial x} + \varepsilon_r v \frac{\partial v}{\partial r} = R_r - \frac{\varepsilon_r}{\rho} \frac{\partial p}{\partial r} + \nu_{ef}\left[\varepsilon_x \frac{\partial^2 v}{\partial x^2} + \varepsilon_r\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right)\right], \tag{1.1}$$

$$\rho c_p \varepsilon \frac{\partial T}{\partial \tau} + \rho c_p\left(\varepsilon_x u \frac{\partial T}{\partial x} + \varepsilon_r u \frac{\partial T}{\partial r}\right) = Q + \lambda_{ef}\left[\varepsilon_x \frac{\partial^2 T}{\partial x^2} + \varepsilon_r\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right)\right].$$

Here A characterizes the action of the volume forces that occur during temperature expansion of the fluid. Within the framework of the Boussinesq model $A = g(T - T_*)/T_*$ (T_* is a certain characteristic temperature and g is the free fall acceleration). The quantities R_x, R_r, Q reflect the force and thermal interaction of the rods with the fluid per unit volume, and the parameter G is mass liberation or absorption which can occur in the system because of chemical reactions or gas liberation on the rod surfaces. The equations (1.1) describe the distributions of the true stream parameters averaged over the liquid volume. Introduction of the concept of the filtration velocity $\mathbf{V}_1 = \varepsilon \mathbf{V}$ permits extension of these equations to the whole volume under consideration. To complete the formulation they must be supplemented by the heat transport equation in the solid phase, particular cases of which can be conditions of isothermy T_c = const or heat liberation q_c = const on the rod surfaces.

The equations obtained contain the geometric characteristics ε, ε_x, ε_r of the space and the dynamic phase interaction parameters R_x, R_r. In the general case of a flow in an anisotropic porous structure $\varepsilon_x \neq \varepsilon_r$ and $R_x \neq R_r$. However, taking into account that $\varepsilon_x = \varepsilon$ for a bundle of rods of constant radius, and introducing the additional assumption that $\varepsilon_r \sim \varepsilon$ also, we can simplify (1.1) after which it will agree completely with the equations in [11].

The following possibility of simplifying the initial equations includes utilization of an approximate boundary layer model [12]. Assuming the Rayleigh number for the bundle to be considerably larger than one and that the transverse velocities are significantly less than the longitudinal, we obtain the following system (the flow is stationary, $p = p_\infty$, $dp/dx = 0$, $T_* = T_\infty$ = const):

$$\varepsilon^{-1}\left(u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial r}\right) = f + \varepsilon g \frac{T_1 - T_\infty}{T_\infty} + \nu_{ef}\left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r}\frac{\partial u_1}{\partial r}\right), \tag{1.2}$$

$$\rho c_p \varepsilon^{-1}\left(u_1 \frac{\partial T_1}{\partial x} + v_1 \frac{\partial T_1}{\partial r}\right) = \varepsilon^{-1} q + \lambda_{ef}\left(\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r}\frac{\partial T_1}{\partial r}\right), \quad \frac{\partial (ru_1)}{\partial x} + \frac{\partial (rv_1)}{\partial r} = G.$$

Let us note that (1.1) and (1.2) contain certain provisional parameters μ_{ef} and λ_{ef} in place of the physical viscosity and heat conduction of the fluid, which can be used to take account a different kind of inaccuracy in the averaging and the influence of solid surfaces on dissipative processes. However, as is shown in [13, 14], $\mu_{ef} = \mu$ can be assumed with great accuracy for filtration flows. This condition is also assumed for λ_{ef} in this paper.

A domain of moving homogeneous fluid is contiguous to the outside of the bundle. Since the flow is considered within the framework of the boundary layer model in the inner domain, then it is also completely natural to use the axisymmetric boundary layer equations in the other domain, which differ from (1.2) by the fact that $\varepsilon = 1$, $f = q = G = 0$ and the subscript 2 is used instead of 1.

Hydrodynamic and thermal interaction exists between the two domains. It is reflected formally by using equality of the velocities, temperatures, stresses, and heat fluxes on the boundary of the viscous and filtration flow [15]

$$u_2(x, R_b) = \varepsilon^{-1}u_1(x, R_b),\ v_1(x, R_b) = v_2(x, R_b),$$
$$T_1(x, R_b) = T_2(x, R_b), \tag{1.3}$$
$$\mu \frac{\partial u_2}{\partial r}\Big|_{r=R_b} = \mu_{ef} \frac{\partial u_1}{\partial r}\Big|_{r=R_b},\quad \lambda \frac{\partial T_2}{\partial r}\Big|_{r=R_b} = \lambda_{ef} \frac{\partial T_1}{\partial r}\Big|_{r=R_b}.$$

Besides these relationships, conditions on the bundle axis and on the boundaries of the whole flow domain are still needed

$$v_1 = 0,\ \partial u_1/\partial r = 0,\ \partial T_1/\partial r = 0 \ \text{for } r = 0,$$
$$u_2 \to 0,\ T_2 \to T_\infty \qquad \text{for } r \to \infty. \tag{1.4}$$

For completion of the set problem it remains to determine f and q.

2. DETERMINATION OF THE VOLUME DRAG AND HEAT LIBERATION IN A BUNDLE OF RODS

Let us use the model of a "free cell" for two-phase systems for this purpose [16, 17]. The cell is the domain between two coaxial cylinders for the case of a longitudinally streamlined cylinder that is part of the rod assemblage. The inner cylinder is a streamlined cylindrical body with radius R_c and the outer is a liquid shell of radius R_Δ. The radius of the outer cylinder is taken such that the ratio between the fluid volume in the space between rods and the total volume of the bundle would equal the porosity, i.e., $\varepsilon = 1 - (R_c/R_\Delta)^2$. The equations describing the stabilized fluid motion and the rod heat transfer are solved in the model taken by using symmetry conditions on the cell outer boundary. Consequently, the rods in the bundle are insulated and do not interact with each other.

Such an approach does not permit reflection of the influence of the boundary conditions of the whole process on the distribution of the main parameters over the bundle thickness. In this connection, we modify the model under consideration in such a manner as to remove this constraint.

We will assume that the flow in the space between the rods is free convective, axisymmetric, and described in the approximations of the boundary layer and Boussinesq models by the equations

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = g\left(\frac{T}{T_\infty} - 1\right) + \frac{\nu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right),$$
$$\rho c_p\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r}\right) = \frac{\lambda}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) \tag{2.1}$$

with the boundary conditions

$$u = u_\Delta,\ T = T_\Delta \ \text{for } r = R_\Delta,\ u = 0,\ T = T_c \ \text{or}$$
$$-\lambda \partial T/\partial r = q_c \ \text{for } r = R_c \tag{2.2}$$

(U_Δ, T_Δ are certain provisional quantities that should be defined in terms of the stream filtration parameters). In such a representation the cells comprising the bundle are already not insulated and are connected with the bundle parameters and the external boundary layer by means of the functional dependences $U_\Delta = F_u(u_1, T_1)$, $T_\Delta = F_T(u_1, T_1)$. We use successive approximations [18] to solve (2.1), where the solution is represented in the form $u = u_0(x, r) + u_1(x, r) + \ldots$ and the function u_0 satisfies the simplified equation that can be obtained from (2.1) by omitting the convective components, i.e.,

$$\frac{\nu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u_0}{\partial r}\right) + g\left(\frac{T_0}{T_\infty} - 1\right) = 0,\quad \frac{\lambda}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T_0}{\partial r}\right) = 0 \tag{2.3}$$

(we later omit the subscript 0 since we shall use only the zeroth approximation).

We take (2.2) as boundary conditions for (2.3). Formulation of this problem and certain computation results are presented in [19] for a bundle of isothermal rods.

The expressions

$$T = T_\Delta + T_q(\ln R_\Delta/R_c - \ln r/R_c),$$
$$u = U_\Delta \frac{\ln r/R_c}{\ln R_\Delta/R_c} + \frac{1}{4}\mathrm{Gr}^*\left(\frac{\nu}{R_b}\right)\frac{1}{R_b^2}\left[(r^2 - R_\Delta^2)\ln r/R_c + \right.$$

$$+\left(\ln R_\Delta/R_c+1+\frac{T_\Delta-T_q}{T_q}\right)(R_\Delta^2\varepsilon+R_c^2-r^2)\Bigg] \tag{2.4}$$

[T_q is the characteristic temperature of the rod, $T_q = q_cR_c/\lambda$, Gr* is a modified Grashof number $Gr^* = gR_b{}^3T_q(\nu^2T_\infty)$] are the solution of the system (2.3) with the boundary conditions q_c = const. By having the distributions of u and T in the cell the friction and heat flux on the rod can be determined and related to the desired f and q:

$$f=\frac{2\pi R_c}{\pi R_\Delta^2}\,\mu\,\frac{\partial u}{\partial r}\bigg|_{r=R_c},\quad q=\frac{2\pi R_c}{\pi R_\Delta^2}\,\lambda\,\frac{\partial T}{\partial r}\bigg|_{r=R_c}. \tag{2.5}$$

Using (2.4) in (2.5), we obtain

$$f=\frac{2\nu U_\Delta}{R_\Delta^2\ln R_\Delta/R_c}+g\frac{T_q}{T_\infty}\left[(\varepsilon-1+a_\varepsilon)\left(\ln R_\Delta/R_c+\right.\right.$$
$$\left.\left.+\frac{T_\Delta-T_\infty}{T_q}\right)+a_\varepsilon-1+\varepsilon/2\right], \tag{2.6}$$
$$q=2\lambda T_q/R_\Delta^2\quad(a_\varepsilon=\varepsilon/(2\ln R_\Delta/R_c)).$$

Let us now make the following assumption. We consider the filtration velocity equal to the mean mass flow rate in the cell under consideration, and the filtration temperature to be the mean calorimetric value of the flux passing the space between the rods

$$u_1=2\pi\int_{R_c}^{R_\Delta}urdr/(\pi R_\Delta^2),\quad T_1=2\pi\int_{R_c}^{R_\Delta}uTrdr/(\pi R_\Delta^2u_1). \tag{2.7}$$

After substituting (2.4) into (2.7) and appropriate manipulations, we find the relation between U_Δ, T_Δ, and u_1, T_1:

$$U_\Delta=AU_q+BU_q(T_\Delta/T_q),\ T_\Delta=(u_1T_1-U_qT_q)/A_\Delta, \tag{2.8}$$

where

$$AU_q=\frac{u_1+Gr^*(\nu/R_b)\left[(\ln R_\Delta/R_c-T_\infty/T_q+1)\,b_1+b_2\right]}{1-a_\varepsilon};$$

$$BU_q=Gr^*(\nu/R_b)\frac{b_1}{1-a_\varepsilon};\ A_\Delta=u_1-Gr^*(\nu/R_b)\left[\frac{b_1b_3}{1-a_\varepsilon}+\left(\frac{R_\Delta}{R_b}\right)^2\frac{\ln R_\Delta/R_c}{8}b_4\right];$$
$$U_q=u_1\ln R_\Delta/R_c-AU_qb_3-Gr^*(\nu/R_b)(R_\Delta/R_b)^2(\ln R_\Delta/R_c)/8\times$$
$$\times[b_5+(1+\ln R_\Delta/R_c-T_\infty/T_q)b_4].$$

The set of constants b_1, ..., b_5 is determined only by the bundle geometry: $b_1=(R_\Delta/R_b)^2\times\varepsilon(a_\varepsilon+\varepsilon/2-1)/4$, $b_2=(R_\Delta/R_b)^2\ln R_\Delta/R_c(1-a_\varepsilon-\varepsilon a_\varepsilon/2)/8$, $b_3=\ln R_\Delta/R_c-1+a_\varepsilon$, $b_4=4a_\varepsilon b_3+(1{,}5\varepsilon-1)\times a_\varepsilon-2\varepsilon+1$, $b_5=1{,}5(1-a_\varepsilon)-\ln R_\Delta/R_c-\varepsilon a_\varepsilon/4$.

Consequently, the conjugate problem of free convective heat transfer of an open bundle of vertical rods consisting of the filtration flow equations in the bundle (1.2), the external boundary layer equations (f = q = 0, ε = 0), and the boundary conditions (1.4) has been formulated. Additional dependences for the friction and the volume heat liberation in a bundle of heat liberating rods are given by the relationships (2.6) and (2.8).

3. RESULTS OF COMPUTATIONS, COMPARISON WITH EXPERIMENT

We apply the numerical method described in [20] to solve the formulated problem.

One of the few papers containing an exposition of the theoretical and experimental investigations of the heat transfer of a bundle of heat liberating rods is [8]. The mathematical description of the process therein is carried out by using the cell model with boundary conditions of the free surface type, which excludes the influence of the external medium on the heat transfer in the bundle. Conditions at the center will naturally be closest to these conditions for an axisymmetric bundle interacting with the external medium. Consequently, we select rod parameters at the center of the bundle for comparison of the results. The triangular stacking scheme in [8] consisting of 42 rods of radius 0.0079 m determined the dependence between the porosity and the relative spacing

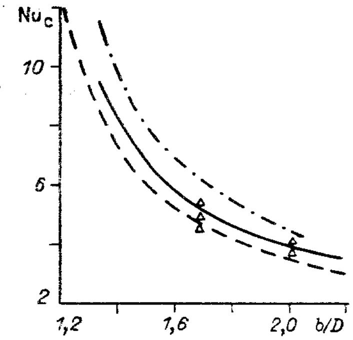

Fig. 2

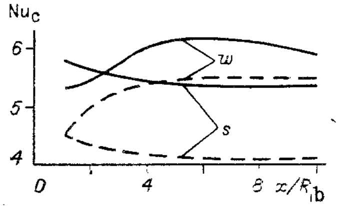

Fig. 3

$$\varepsilon = 1 - \pi/(2\sqrt{3})(D/b)^2 \tag{3.1}$$

[b is the distance between centers, D is the rod diameter, $R_b = R_c(\sqrt{2\sqrt{3}N}/\sqrt{\pi})(b/D)$ (N is the quantity of rods in the bundle)].

The thermophysical parameters of the medium corresponded to the parameters for air $\rho = 1.21$ kg/m³, $c_p = 1005$ J/(kg·K), $\nu = 0.15 \cdot 10^{-4}$ m²/sec, Pr = 0.7, $T_\infty = 20°C$. In the computations b/D = 1.4-2.2 and q_c = 125-500 W/m² were varied and this corresponds to the range of variations of the governing parameters in [8].

By using (2.4) the value of the local Nusselt number for a rod in the cell is

$$\mathrm{Nu}_c = \frac{\alpha D}{\lambda} = \frac{2}{T_c - T_1} = \frac{2}{T_\Delta + \ln R_\Delta/R_c - T_1}$$

(α is the heat elimination coefficient and T_c is the rod surface temperature).

The values of Nu_c at the center of the bundle, obtained by the method proposed, and compared in Fig. 2 with theoretical (dash—dot) and experimental (triangles) results [8]. The good agreement between the distributions found and the available experimental data for b/D = 1.68 and 2.03 should be noted, which permits making a deduction about the adequacy of the proposed heat transfer model and the computation methodology.

As computations show, the influence of the external medium is felt in a zone of small thickness on the bundle surface. Elevation of the heat transfer intensity here is due to the formation of a section of external cold gas ejection because of the flow development in the volume of the bundle. This results in an increase in Nu_c and a reduction of the rod temperature on the surface as compared with the center. Distributions of Nu_c on the surface (w) and at the center (s) of bundles with heat flux q_c = 125 W/m² on each rod are represented in Fig. 3. The relative spacing is b/D = 1.68 (solid curves) and 2.03 (dashes).

Rod temperature distributions T_c for the modifications considered are displayed in Fig. 4 (the provisional notation agrees with the notation in Fig. 3). A further increase in q_c (or Gr*) results in substantial magnification of the ejection. The transverse flow velocities in the outer boundary layer and on the bundle surface can here reach values commensurate with the longitudinal velocities. Application of the boundary layer model for the filtration flow in the bulk of the rod bundle becomes without foundation in such regimes. It is then necessary to go over to solving the problem in the complete formulation with the dynamic and thermal phase interaction taken into account during transverse flow around the rods.

Free-convection flow in a bundle of isothermal rods was also examined in this paper. As follows from [19], in this case

$$\mathrm{Nu}_c = \frac{2}{T_c - T_1} \frac{T_c - T_\Delta}{\ln R_\Delta/R_c}, \tag{3.2}$$

while the equivalent Gr* of the modified Grashof number is $\mathrm{Gr} = gR_b^3(T_c - T_\infty)/(\nu^2 T_\infty)$. As computations showed, the $\mathrm{Nu}_c(x)$ distributions for bundles of heat liberating and isothermal rods with identical geometric characteristics are practically in agreement if the condition Gr = Gr* is satisfied. A change in the heat liberation intensity or the initial temperature, which is identical to the change in the numbers Gr* or Gr, is not felt in the rod heat transfer in the central part of the bundle in practice. This confirms the deduction in [8] that the density of their stacking exerts governing influence on the rod heat transfer intensity. Let us analyze this fact in an example of an isothermal bundle.

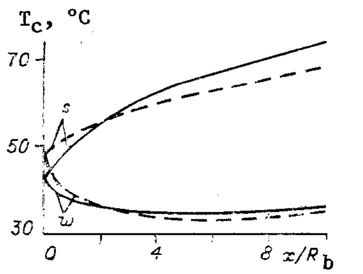

Fig. 4

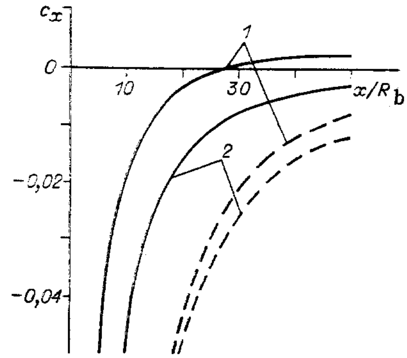

Fig. 5

Let us consider the rod heat transfer in a cell for the limit case when thermal stabilization sets in the central domain of the bundle. Let us take the following stabilization condition $T_c - T_1 \le 10^{-2} T_c$. The Nu_c for this flow is calculated from (3.2) while the equation determining T_Δ is presented in [19]. The solution of the equation is written as

$$T_\Delta - T_c = (-BT + BT\sqrt{1-\beta})/(2AT), \tag{3.3}$$

where

$$AT = (\nu/R_b)\frac{Gr}{(T_c - T_\infty)\ln R_\Delta/R_c}\left[c_3 + c_5 + \frac{c_4(c_1+c_2)}{1-a_\varepsilon}\right];$$

$$BT = AUc_4 - (\nu/R_b)\,Gr\,c_5;\quad AU = \frac{u_1 - (\nu/R_b)\,Gr\,c_1}{1-a_\varepsilon};$$

$$c_1 = -b_1;\ c_2 = -b_2;\ c_3 = (R_\Delta/R_b)^2[\ln R_\Delta/R_b + 1{,}5(a_\varepsilon - 1) + \varepsilon a_\varepsilon/4]/8;$$

$$\beta = 4ATu_1(T_c - T_1)/BT^2;\ c_4 = [\ln R_\Delta/R_c - 1 + a_\varepsilon]/\ln R_\Delta/R_c;$$

$$c_5 = (R_\Delta/R_b)^2[(1-a_\varepsilon)(2\varepsilon - 1) + \varepsilon a_\varepsilon/2 - 2\varepsilon c_4]/8.$$

The thermal stabilization is $\beta \sim 10^{-2}$ under the condition taken. Consequently, by linearizing (3.3) with respect to β and appropriate manipulations we have

$$T_\Delta - T_c = -\frac{u_1(T_c - T_1)}{u_1 c_4/(1-a_\varepsilon) + (\nu/R_b)\,Gr\,c_0}. \tag{3.4}$$

Then taking (3.4) into account

$$Nu_c = \frac{1}{\ln R_\Delta/R_c}\,\frac{2}{c_4/(1-a_\varepsilon) + (\nu/R_b)\,Gr\,c_0/u_1} \tag{3.5}$$

($c_0 = b_1c_1/(1 - a_\varepsilon) - c_5$, $u_1/(\nu/R_b)$ is the dimensionless filtration flow velocity in the bundle).

As is seen, the second component in the denominator of (3.5) takes account of the contribution of the flow dynamic parameters to the rod heat transfer. Analysis shows that the constant c_0 depends only on b/D or ε as varies between the limits $(-0.0018$ to $-0.0032)(R_\Delta/R_b)^2$ as b/D changes from 1.2 to 2.2. If it is assumed that the beam consists of 100 rods, say, then for $Gr = 10^5$ the second component is equivalent to $B(\nu/R_b)/u_1$, where $B \sim 1$.

Therefore, the influence of the stream dynamic parameters on Nu_c in the cell decreases in proportion to $1/u_1$, i.e., flux transformation from free convective to forced occurs during the flow development, which corresponds to flow around and heat transfer to a rod in a channel with axial parameters U_Δ, T_Δ. Omitting the dynamic component in (3.5), the lower limit value of Nu_c^* is written in the form

$$Nu_c^* = \frac{-2[\ln(1-\varepsilon) + \varepsilon]}{\ln(1-\varepsilon)[0{,}5\ln(1-\varepsilon) + 1] + \varepsilon} \tag{3.6}$$

[the porosity ε is calculated from (3.1)]. The distribution $Nu_c^*(\varepsilon)$ found is represented by dashed lines in Fig. 2. The maximal deviation of the values of Nu_c^* from the numerical results obtained by the method described above does not exceed 11%. This permits the recommendation of the approximate dependence (3.6) as the lower bound of the heat transfer intensity during development of the free convection in a rod bundle.

Attention was turned in [19] to the existence of two characteristic free convection flow modes in the outer boundary layer on a bundle of isothermal rods. The first is realized in the section from the beginning of filtration flow formation to the onset of thermal equilibrium between the gas and the rods in the bundle. The flow in the outer domain is here analogous to forced flow with suction on an accelerating cylinder of radius R_b. For this flow $c_x < 0$ ($c_x = 1/(\rho u_0{}^2)\mu(\partial u/\partial r)|_{r=R_c}$, and u_0 is the initial velocity in the bundle). A second flow mode is developed in the external flow after thermal equilibrium has been achieved in the bundle. The velocity profile in the boundary layer acquires a characteristic form for free convective flow with $c_x > 0$. The condition $c_x = 0$ is a criterion for the passage from one flow mode to the other.

The distributions $c_x(x)$ for bundles of isothermal rods are represented in Fig. 5 (solid lines). It is assumed that a bundle of radius 0.02 m consists of a set of rods of radius 0.00025 m heated to a temperature $T_c = 200°C$. The temperature of the external medium is $T_\infty = 20°C$, and the Prandtl number is $Pr = 0.7$. The curves 1 correspond to a bundle of 100 rods ($N = 100$) while $N = 50$ for the curves 2. It can be seen that an increase in the stacking density of the bundle results in diminution of the section with the first flow mode. Distributions c_x are given here for bundles of heat liberating rods (dashed lines). The condition $Gr = Gr^*$ is satisfied here. For the heat transfer modifications considered $c_x < 0$ in the whole computation interval, i.e., the second flow mode in the boundary layer does not develop in such a bundle, which can be explained by the absence of the limit value of the rod temperature for a bundle with heat liberation unlimited along the length.

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MODIFICATION OF THE METHOD OF DISCRETE CONTINUATION BY PARAMETERS

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We consider a system of nonlinear equations

$$F_i(x_1, x_2, \ldots, x_n, p) = 0 \quad (i = \overline{1, n}), \tag{1}$$

where x_i $(i = \overline{1, n})$ are arguments and p is the solution parameter. Nonlinear problems of mechanics can often be reduced to systems of this kind. One such elementary problem is the problem of axisymmetric buckling of an isotropic circular plate acted upon by radial forces N_0 distributed uniformly on the contour and by a transverse load q:

$$\begin{gathered} r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r^2N_r)\right] + \frac{Eh}{2}\left(\frac{dw}{dr}\right)^2 = 0, \\ \frac{D}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} - \frac{1}{r}\frac{d}{dr}\left(rN_r\frac{dw}{dr}\right) = q \quad (0 \leqslant r \leqslant R), \\ dw/dr = Q_r = u_r = 0 \text{ for } r = 0,\ w = M_r = 0,\ N_r = -N_0 \text{ for } r = R. \end{gathered} \tag{2}$$

Here, u_r and w are the radial displacement and the deflection; N_r, M_r, and Q_r are the specific radial force, the bending moment, and the shearing force; E and D are Young's modulus and the cylindrical rigidity of the plate; R and h are the plate's radius and thickness, respectively.

We propose to construct the loading trajectory of a mechanical object whose behavior is described by system (1):

$$x_i = x_i(p)\ (i = \overline{1, n}).$$

The method of continuous parameter continuation is convenient for solving this problem. With this method one constructs the loading trajectory at all points that are regular in the Poincaré sense, including the limiting points of the trajectory. The idea of this method was first advanced in [1]. Its detailed elaboration, which considers the equivalence of solution variables, was given in [2]. However, this method has a shortcoming: In the course of numeric construction of the loading trajectory, an uneliminable error accumulates in the solution. After several steps in the continuous continuation method, one has to adjust its solution. This adjustment is done by an algorithm that relies on the techniques of the method of discrete continuation in the parameter, which also implements the concept of equivalence of parameters [2]. On this basis, one can adjust the solution at regular and limiting points of the trajectory. Without reviewing the various methods of continuous and discrete continuation (such a review can be found in [2, pp. 12-23, 176-196]), we will examine

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